Further Maths Unit Overview Year 12

Further Maths – Year 12 Autumn Term 1				
What are we learning?	**What knowledge, understanding and skills will we gain?**	**What does mastery look like?**	**How does this build on prior learning?**	**What additional resources are available?**
Core 1 - Complex numbers, Argand diagrams, Series (Ch1-3)	Knowledge: Understand the real and imaginary parts of a complex number a+bi Understand and manipulate complex numbers Find complex solutions to quadratic equations Find complex solutions to cubic and quartic equations Represent complex numbers on an Argand diagram Put a complex number in modulus-argument form Identify loci and regions Maximise and minimise the modulus and argument of z Recall and use series notation Find the sum of the first n natural numbers Break up summations in order to evaluate them more easily Understanding: Complex roots of polynomials with real coefficients come in conjugate pairs. Skills: Calculations with complex numbers Algebraic manipulation Apply the formulae for the sum of the first n integers, first n square numbers and first n cube numbers	In particular students should: Be able to confidently factorise and solve polynomials up to and including quartics Be able to answer more complex problems by applying their knowledge of loci and co-ordinate geometry Be able to sketch regions on an Argand diagram Be able to find minimum and maximum values Use known summations results where possible and partition problems into smaller summations	GCSE Higher – Rationalising a denominator Factorising a quadratic equation Loci Co-ordinate geometry – gradient, parallel and perpendicular lines, circle geometry, perpendicular bisectors Pythagoras Theorem C1 - Factor Theorem Dividing complex numbers using conjugates is comparable to dealing with surds at GCSE and covered in 12 Maths Aut 1 also.	Core Pure 1Textbooks SoL – guidance for each individual lesson Resources – outline Powerpoints with suggested examples and scaffolding activities Chapter assessments and practice using past exam questions For extension use: UKMT senior challenge, MAT and STEP Foundation materials

Further Maths - Year 12 Autumn 2				
What are we learning?	**What knowledge, understanding and skills will we gain?**	**What does mastery look like?**	**How does this build on prior learning?**	**What additional resources are available?**
Core 1 - Roots of polynomials, matrices, linear transformations (Ch4,6,7)	Knowledge: Derive and use the relationship between the roots of quadratic, cubic and quartic equations. Evaluate expressions relating to the roots of polynomials. Understand the concept of a matrix, define zero and identity matrices, multiply as matrix by a scalar, multiply matrices, calculate the determinant of a matrix, transpose a matrix. Understanding: Find the equation of a polynomial whose roots are a linear transformation of the roots of a given polynomial. Use and derive expressions relating to reciprocals and products of powers. Find the inverse of a matrix, use matrices to solve equations, interpret simultaneous equations geometrically. Represent linear transformations using matrices. Understand the connection between the determinant and the scale factor for the change of area in an enlargement. Perform successive transformations using matrices. Perform linear transformations in 3 dimensions Skills: Expanding polynomials, factorising expressions, algebraic manipulation, model real situations using matrices, write down the matrix representing a rotation about any angle.	In particular students should: Be able to identify the connection between the rules for roots for different order polynomials as a way of remembering them. Given a polynomial equation, find the equation of polynomials with roots which are a linear transformation of the originals. Given three roots of a quartic, be able to identify the fourth root using the roots of polynomials method. Be able to explain the conditions required when multiplying matrices, why the order matters when performing a calculation. Given a determinant, and other conditions, students will be able to find an unknown in a matrix. Describe a transformation represented by a matrix geometrically.	Students will need to use the fact that roots of polynomials with real coefficients come in conjugate pairs and to be able to Illustrate roots on an argand diagram from autumn term 1. Knowledge of GCSE transformations, trigonometry, and Pythagoras.	Core Pure 1Textbooks, in particular mixed and review exercises. SoL – guidance for each individual lesson Resources – outline Powerpoints with suggested examples and scaffolding activities Chapter assessments and practice using past exam questions For extension use: UKMT senior challenge, MAT and STEP Foundation materials

Further Maths - Year 12 Spring 1				
What are we learning?	**What knowledge, understanding and skills will we gain?**	**What does mastery look like?**	**How does this build on prior learning?**	**What additional resources are available?**
Core 1 - Proof by induction, vectors and scalar product, volumes of revolution (Ch 8,9,5)	Knowledge: Understand the principle of proof by induction and prove results about sums of series, divisibility and about matrices. Understand and use vector and cartesian forms of the equation of a straight line in 3D, understand and use the vector and cartesian forms of the equation of a plane, calculate the scalar product of two 3D vectors, calculate the angle between two vectors, two lines, a line and a plane, or two planes. Find the volume of revolution when a curve is rotated around the x or y axis, Understanding: Identify the steps in the induction process, understand and use the scalar product form of the equation of a plane, Determine whether two lines meet and determine the point of intersection, calculate the perpendicular distance between: two lines, a point and a line, or a point and a plane. Find more complicated volumes of revolution, model real-life objects using volumes of revolution Skills: manipulating expressions using index laws, definite integration.	In particular students should: • Identify an error made in a proof by induction. • Identify the correct form of the equation of a line or a plane top use to solve a problem • Determine when two lines are skew • Find the volume of a solid formed from two curves, or a curve and a straight line Find the volume of a real -life object modelled by a curve, or curves. Identify limitations of the model.	Autumn Term 1 A level Maths sequence notation Rules of indices GCSE and Autumn term 1 foundations of A level Matrices work from Autumn term 2 Further Maths Vectors in 2D covered at GCSE and taken further in Maths A level in Spring 1. Integration concepts Spring 1 Maths A level	Core Pure 1Textbooks, in particular mixed and review exercises. SoL – guidance for each individual lesson Resources – outline Powerpoints with suggested examples and scaffolding activities Chapter assessments and practice using past exam questions For extension use: UKMT senior challenge, MAT and STEP Foundation materials

Further Maths - Year 12 Spring 2				
What are we learning?	**What knowledge, understanding and skills will we gain?**	**What does mastery look like?**	**How does this build on prior learning?**	**What additional resources are available?**
D1 - Algorithms (sorting, bin-packing, order of, Prim's, Kruskal's, Dijkstra's, route inspection), linear programming, critical path analysis	Knowledge: Bubble sort, quick sort and the three bin-packing algorithms. Terminology used in graph theory. Kruskal's and Prim's algorithms. Dijkstra's algorithm. Route inspection algorithm. Formulations of a linear programming problem. Precedence tables. Identify and know how to calculate early and late event times, identify critical values and calculate the total float of an activity. Gantt charts Understanding: Use and understand algorithms given in words and flow charts. Understand the strengths and limitations of different sorting methods. Understand when a linear programming problem requires integer solutions. Skills: Setting up and solving simultaneous equations. Use matrices to represent graphs and networks. Determine whether a graph is Eulerian, semi-Eulerian or neither using the order of nodes. Illustrate a two-variable linear programming problem graphically. Determine a feasible region and locate an optimal point using an objective line or vertex testing.	In particular students should: Strictly adhere to the steps needed to carry out a sort Be familiar with all the terminology related to graphs and networks, such as valency, subgraph, loop. Determine by inspection the best starting point on a graph Determine the order of Prim's algorithm Introduce variables for unknown weights and form equations Be familiar with and use known conventions, such as leaving a feasible region unshaded Ensure their answer makes sense within the context of a question Know when it is appropriate to make assumptions and what those assumptions are	Related topics in GCSE Higher include: Solving inequalities Graphing inequalities Shading feasible regions Setting up and solving simultaneous equations Computer Science students will be familiar with the idea of algorithms and 'trace tables' as used in Ch1.	Decision 1 Textbook (AS Sections only) SoL – guidance for each individual lesson Resources – outline Powerpoints with suggested examples and scaffolding activities Chapter assessments and practice using past exam questions For extension use: UKMT senior challenge, MAT and STEP Foundation materials

Further Maths - Year 12 Summer 1				
What are we learning?	**What knowledge, understanding and skills will we gain?**	**What does mastery look like?**	**How does this build on prior learning?**	**What additional resources are available?**
FP1 - vectors (vector product), conic sections (parabolas and rectangular hyperbolas), inequalities, t-formulae, numerical methods to solving differential equations	Knowledge: Definition of vector product and ways to calculate it. Use of parametric equations. Equations of parabolas and rectangular hyperbolas. Foci and directrices. Effect on inequalities when multiplying by a possible negative value, and to use squaring to avoid this. New trigonometric functions sec, cosec, cot. The t-formulae. Euler's formulae for approximating solutions to first- and second-order differential equations. The midpoint formulae for approximating solutions to first-order differential equations. Understanding: Cyclical nature of the scalar triple product. Loci resulting from points from conic sections. Properties of focus and directrix for a parabola. Selecting and manipulating the appropriate formula to approximate solutions to differential equations. Skills: Using vector product to calculate areas and volumes. Be able to calculate tangents and normal to conic sections. Using graphs to solve inequalities. Solve equations and prove identities using the t-formulae. Find approximate solutions to differential equations using numerical methods.	Quick recall of the required formulae (eg t-formulae) Have a range of techniques to solve modelling problems (especially loci problems for conics) and being prepared to try alternatives if needed. Be able to adapt standard methods to use the information given in a particular case (eg when using numerical methods to find solutions to differential equations). Students will confidently apply calculus techniques to a wider range of problems (eg finding tangent to a parabola)	Related topics in GCSE Higher include: Vectors Drawing graphs of polynomials Inequalities Trigonometry The vectors work in Ch1 builds directly on the work from Pure1 Ch11 (Maths - Spring 1) and Core1 Ch9 (Spring 1). Finding tangents and normals through differentiation is seen in Pure1 Ch12 (Maths – Spring 1)	Further Pure 1 Textbook (AS Sections only) SoL – guidance for each individual lesson Resources – outline Powerpoints with suggested examples and scaffolding activities Chapter assessments and practice using past exam questions For extension use: UKMT senior challenge, MAT and STEP Foundation materials

Further Maths - Year 12 Summer 2				
What are we learning?	**What knowledge, understanding and skills will we gain?**	**What does mastery look like?**	**How does this build on prior learning?**	**What additional resources are available?**
C3/C4 - advanced techniques in calculus (in preparation for Core 2) (C3 Ch8) (C4 Ch1, 4.1-4.3, 6.1-6.7)	Knowledge: Differentiation techniques: chain rule, product rule, quotient rule. Derivatives of exponentials, logarithms, trigonometric functions, and combinations of these. Format of partial fractions for denominators with linear and repeated factors. Implicit differentiation. Integration techniques: reversing the chain rule, using standard patterns, using partial fractions, substitution, by parts. Understanding: When to use different calculus methods (eg chain rule vs product rule). Link of implicit differentiation to the chain rule. Link of integration by parts to the product rule. Link of partial fractions to 'reversing' addition of fractions. Skills: Be able to differentiate and integrate a much wider range of expressions. Be able to decompose a fraction into partial fractions, using algebraic division if necessary. Be able to use trigonometric identities to manipulate integrands into formats ready for integration.	Usually selects the correct technique to tackle a calculus problem, but is prepared to try alternatives when needed. Can confidently manipulate fractions to generate partial fractions, using division if necessary. Being able to use trigonometric formulae to manipulate expressions to be more easily integrated. Students will confidently apply a wider range of calculus techniques to problems.	The differentiation work builds directly on techniques in Pure 1 Ch12 (Maths – Spring 1). The integration work builds directly on techniques in Pure 1 Ch13 (Maths – Spring 1) Partial fractions may have already been taught in (Maths – Summer 2) and only need revision. Use of trigonometric identities builds on Pure 1 Ch10 (Maths – Autumn 2) but also the newer trigonometric ratios seen in FP1 Ch5 (Summer 1)	C3 and C4 Textbooks SoL – guidance for each individual lesson Resources – outline Powerpoints with suggested examples and scaffolding activities Chapter assessments and practice using past exam questions For extension use: UKMT senior challenge, MAT and STEP Foundation materials (although the mixed integration work is particularly challenging initially and students should prioritise ensuring they are comfortable with this first)